On the influence of resonance photon scattering on atom interference

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Abstract. Here, the influence of resonance photon-atom scattering on the atom interference pattern at the exit of a three-grating Mach-Zehnder interferometer is studied. It is assumed that the scattering process does not destroy the atomic wave function describing the state of the atom before the scattering process takes place, but only induces a certain shift and change of its phase. We find that the visibility of the interference strongly depends on the statistical distribution of transferred momenta to the atom during the photon-atom scattering event. This also explains the experimentally observed (Chapman *et al* 1995 *Phys. Rev. Lett.* **75** 2783) dependence of the visibility on the ratio $d_p/\lambda_i = y'_{12}(2\pi/kd\lambda_i)$, where y'_{12} is distance between the place where the scattering event occurs and the first grating, k is the wave number of the atomic center-of-mass motion, d is the grating constant and λ_i is the photon wavelength. Furthermore, it is remarkable that photon-atom scattering events happen experimentally within the Fresnel region, i.e. the near field region, associated with the first grating, which should be taken into account when drawing conclusions about the relevance of "which-way" information for the interference visibility.

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1. Introduction

With the rise and advancement of neutron [1] and atom interferometry [2, 3], it has become feasible the realization of the well-known *gedanken* experiments devised by Einstein during his famous discussions with Bohr [4], later also considered by Feynman [5]. These discussions were focused on the understanding and interpretation of the wave-particle duality and, therefore, the completeness of Quantum Mechanics. In particular, Bohr [4] and Feynman [5] argued that wave and particle properties were complementary, i.e. they could not be simultaneously observed experimentally. On the other hand, aimed to disprove the concept of *complementarity*, Einstein devised double-slit type experiments [4] where it should be possible to obtain "which-way" information without influencing the interference pattern. Einstein's viewpoint based on the compatibility of the wave and particle properties, i.e. that both are present simultaneously and, therefore, can be observed in quantum interference experiments, was supported by De Broglie [6] and Bohm [7]. For these authors, the quantum system comprises both a wave *and* a particle, the former guiding the motion (evolution) of the latter, which leads to a hydrodynamic-like view of Quantum Mechanics [8, 9].

More recently, Rauch and Vigier have pointed out [10] that de Broglie's and Bohm's arguments were based on the so-called *einweg* experiments, which should explicitly show that individual particles go along one trajectory, but without evidencing which particular trajectory. Of course, the "which-way" argument implies the *einweg* one, but not vice versa. The difference between these two types of argument arises from the different signatures of wave and particle properties invoked in them. More specifically, for Bohr and Feynman the particle signature is the "which-way" information, while the wave signature is the visibility or relative contrast of the interference pattern. On the contrary, for de Broglie [6], Bohm [7], Philippidis *et al* [11] or Sanz *et al* [12–14], the particle signature is the arrival of individual quantum particles to a screen (array of detectors) and the time evolution of the distribution of these arrivals [15]; the wave signature associated with each quantum particle is the visibility of the interference pattern together with the fact that it comes from the accumulation of arrivals of a large number (theoretically, an infinite number) of atoms, photons, electrons, etc.

To perform experimentally Feynman's *gedanken* experiment, Chapman *et al* [16] scattered single photons from Na atoms within a three-grating Mach-Zehnder atom interferometer. By measuring the transmission of atoms through the third grating, the influence of photon scattering processes (which take place at a distance y'_{12} from the first grating) on the visibility of the atom interference pattern was investigated. These results have intensified a controversial discussion on the wave-particle duality issue. At the time when the experiment was carried out, this controversy evolved towards a discussion around the question: Is complementarity more fundamental than the uncertainty principle? [17–20] This discussion continued [21–25] with the aim to determine the cause of the visibility decrease: Does the visibility decrease arise (a) from a random momentum transfer between the atom and the photon or (b) from the correlations between the "which-way" detector and the atomic motion? More recently, the statement (b) has been reformulated as [3]: Is the visibility decrease (decoherence) the result of entanglement between a quantum system (the atom) and an environment (the emitted photon, which carries information about the atom's path).

Previously, we explained [26] the experiment carried out by Chapman *et al* using the solution of the time-dependent Schrödinger equation for an atom interacting with a photon and the gratings in a three-grating Mach-Zehnder interferometer. In our explanation, wave and particle properties were compatible, since in our opinion both are present and play a role. We derived an analytic expression for the visibility dependence on the ratio $d_p/\lambda_i = y'_{12}(2\pi/kd\lambda_i)$, where k is the wave number of the atomic center-of-mass motion, d is the grating constant and λ_i is the photon wavelength. This theoretical result was in fairly good agreement with the visibility measured in the experiment [16]. The distribution of transferred momentum during the photon scattering process leads to the visibility decrease as d_p/λ_i approaches 0.5 as well as several subsequent revivals with decreasing maxima. Here, we provide additional arguments which support our conclusions of reference [26]. In particular, we study the visibility dependence on the features of the atom selection at the exit of the interferometer, before the detection takes place. We also show that in the experiment the photon-atom interaction takes place within the Fresnel region, i.e. the near field region, associated with the first grating, something that has to be taken into account within any dynamical (i.e. time-dependent) description of the experiment.

2. Wave function of an atom after interacting with a grating and a photon

Consider an initial stationary atomic monochromatic wave, which spreads along the y-axis and is incident to a one-dimensional grating parallel to the x-axis at $y = 0$,

$$\Psi(x, y, t) = e^{-i\omega t}\psi^i(x, y) = B^i e^{-i\omega t} e^{iky}, \qquad y < 0, \tag{1}$$

with B^i being a constant. After interacting with the grating (i.e. after getting diffracted), this incident wave transforms into

$$\Psi(x, y, t) = e^{-i\omega t}\psi(x, y), \tag{2}$$

where

$$\psi(x, y) = \frac{e^{iky}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk_x c(k_x) e^{ik_x x} e^{-ik_x^2 y/2k}, \tag{3}$$

satisfies the Helmholtz equation [27]. If the grating is completely transparent inside the slits (the union of slit areas is denoted by A) and completely absorbing outside them, $c(k_x)$ can be expressed [27] as

$$\begin{aligned} c(k_x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx' \psi(x', 0^+) e^{-ik_x x'} \\ &= \frac{1}{\sqrt{2\pi}} \int_A d_x \psi^i(x', 0^-) e^{-ik_x x'}, \end{aligned} \tag{4}$$

where $\psi^i(x', 0^-)$ and $\psi(x', 0^+)$ denote the wave function just before and just after the first grating, respectively. The solution of the Helmholtz equation, $\psi(x, y)$, given by (3), is equivalent to the Fresnel-Kirchhoff solution

$$\psi(x, y) = \sqrt{\frac{k}{2\pi y}}\, e^{-i\pi/4} e^{iky} \int_{-\infty}^{\infty} dx' \psi(x', 0^+) e^{ik(x-x')^2/2y}. \tag{5}$$

The photon-atom scattering event induced by the laser light at a distance y'_{12} from the first grating leads to a change of the atomic transverse momentum, Δk_x, and, therefore, to a shift of the wave function in the momentum representation. Hence, after an atom absorbs and re-emits again a photon somewhere along the x axis at a time t'_{12} and a distance $y'_{12} = vt'_{12} = (\hbar k/m)t'_{12}$ from the first grating, the atomic wave function takes the form [26]

$$\begin{aligned} \psi_{\Delta k_x}(x, y) = &\frac{e^{iky}}{\sqrt{2\pi}}\, e^{i\Delta k_x (x+\Delta x_0) - i\Delta k_x^2 y/k} \\ &\times \int_{-\infty}^{\infty} dk'_x c(k'_x) e^{-ik'^2_x y/2k} e^{ik'_x(x+\Delta x_0 - \Delta k_x y/k)}, \end{aligned} \tag{6}$$

where

$$\Delta x_0 = \frac{\Delta k_x \hbar t'_{12}}{m} = \frac{\Delta k_x y'_{12}}{k}. \tag{7}$$

The wave function (6), evaluated at the distance y_{12}, which separates the second from the first grating, was used [26] as a wave incident onto the second grating. Then, the wave function propagating towards the third grating was determined using (5), where $\psi(x', 0^+)$ consists of the parts of the incident wave which are transmitted through the slits of the second grating.

This is the way how the evolution of the initial plane wave (1) was determined. After interacting with the first grating, a photon at the distance y'_{12} from this slit,

and a second grating, the resulting wave function evolves freely again up to the third grating. In order to illustrate this evolution, in figure 1 we have plotted the atom probability density within the interferometer at several distances from the first grating. Two values for the transferred momentum are considered: $\Delta k_x = 0$ (blue) and $\Delta k_x = 1.5k_i$ (red). Far from a grating, the straight lines represent the paths along which the maxima of the probability density move; near the grating, these straight lines would just be the prolongation of the paths. Note that in the near field associated with a grating, the wave function has a very complex form and the lines do not exactly represent the paths of the atoms. Within this region, according to a Bohmian picture [13, 14], there are many paths which, as the distance from a grating increases, converge towards three main paths (only two are plotted in figure 1). The extension of the near field is of the order of $10L_T$, where $L_t = 2d^2/\lambda$ is the so-called *Talbot distance* [14].

3. Dependence of visibility on the distribution of transferred momentum

In the experiment [16], the number of Na atoms transmitted through the third grating is counted for the laser off and laser on varying the distance y'_{12} as well as considering different values of the shift Δx_3 of the third grating along the x-axis. In a first round of measurements, all transmitted atoms were collected (counted) without carrying out any selection. Thus, the resulting interference curve can be directly associated with the distribution of transferred momentum, which is given by the Mandel-Wolf expression [28, 29]

$$P_{MW}(\Delta k_x) = \frac{3}{8k_i}\left[1 + \left(1 - \frac{\Delta k_x}{k_i}\right)^2\right]. \tag{8}$$

Then, in subsequent measurements, specific subsets of transmitted atoms were counted after being selected using certain slits positioned after the third grating. As explained in [16], each selection is equivalent to a particular distribution of transferred momentum during the photon-atom scattering process. The experimental data show that the interference pattern visibility strongly depends on the ratio d_p/λ_i as well as on the probability distribution for the transferred momentum, $P(\Delta k_x)$.

In order to explain and interpret these experimental data within our approach [26], it is necessary to study the function

$$T(y'_{12}, \Delta x_3) = \int_0^{2k_i} d(\Delta k_x) P(\Delta k_x) \tilde{T}(y'_{12}, \Delta k_x, \Delta x_3), \tag{9}$$

where

$$\tilde{T}(y'_{12}, \Delta k_x, \Delta x_3) = \int_{slits} |\psi_{\Delta k_x}(x, y = y_{12} + y_{23})|^2 \, dx. \tag{10}$$

The latter function is proportional to the number of atoms transmitted through the third grating that undergone a change of momentum Δk_x during the photon-atom scattering process behind the first grating. By numerical integration, it was shown [26] that (10) has the general form

$$\tilde{T}(y'_{12}, \Delta k_x, \Delta x_3) = a + b\cos(2\pi\Delta x_3/d + d_p\Delta k_x), \tag{11}$$

where a and b are constants, and

$$d_p = (2\pi/kd)y'_{12}. \tag{12}$$

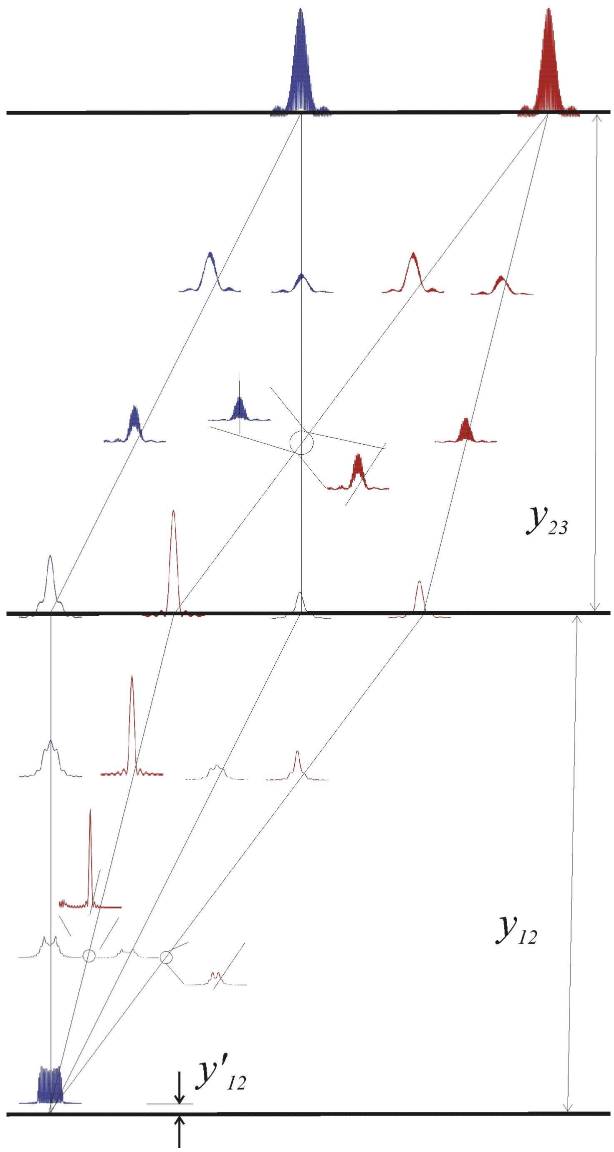

Figure 1. Atom probability density within a three-grating Mach-Zehnder interferometer at several distances from the first grating for two values of the transferred momentum: $\Delta k_x = 0$ (blue) and $\Delta k_x = 1.5k_i$ (red). As in the experiment, the incoming particles are Na atoms and the parameters used are: $v_{\mathrm{Na}} = 1400$ m/s, $k = m_{\mathrm{Na}}v_{\mathrm{Na}}/\hbar = 5.09 \times 10^{11}$ m^{-1}, $k_i = 2\pi/(589$ nm$) = 1.07 \times 10^7$ m^{-1}, $y_{12} = y_{23} = 65$ cm, $d = 200$ nm, slit width $\delta = 100$ nm and number of slits illuminated by the incident atomic plane wave $n = 24$. The distance $y'_{12} = 2.863$ mm corresponds to $d_p/\lambda_i = 0.3$ mm, which lies within the near field region, this being evident from the fact that the Talbot distance is $L_t = 2d^2/\lambda = 6.484$ mm.

For y'_{12} far from a grating, d_p denotes the separation between two paths associated with the zeroth and first order interference maxima; near the grating, it refers to the distance between the prolongations of these paths. Based on these results, we may assume that for certain classes of distribution functions $P(\Delta k_x)$, $T(y'_{12}, \Delta x_3)$ has the general form

$$T(y'_{12}, \Delta x_3) = a + bV \cos(2\pi \Delta x_3/d + \varphi), \tag{13}$$

where the visibility V and phase-shift φ are both functions of the ratio d_p/λ_i, and their particular form is determined by $P(\Delta k_x)$, which is assumed to be normalized on the interval $[0, 2k_i]$.

In order to understand the features of $V(d_p/\lambda_i)$ and $\varphi(d_p/\lambda_i)$, consider, for instance, the uniform distribution $P(\Delta k_x) = 1/2k_i$. The evaluation of the integral of the first term in $T(y'_{12}, \Delta x_3)$ is trivial, and for the second term we have

$$\int_0^{2k_i} \frac{1}{2k_i}\, d(\Delta k_x) b \cos(2\pi \Delta x_3/d + d_p \Delta k_x)$$
$$= \frac{b}{k_i d_p}\, \sin(d_p k_i) \cos(2\pi \Delta x_3/d + d_p k_i). \tag{14}$$

From this result, we reach

$$T(y'_{12}, \Delta x_3) = a + \frac{b}{k_i d_p}\, \sin(d_p k_i) \cos(2\pi \Delta x_3/d + d_p k_i). \tag{15}$$

and, therefore,

$$V = \frac{1}{k_i d_p}\, \sin(d_p k_i) \qquad \text{and} \qquad \varphi = d_p k_i. \tag{16}$$

As can be seen, V vanishes for $d_p k_i = n\pi$, which is easy to explain as follows. The integrand in (14) is a periodic function of Δk_x, with period $2\pi/d_p$. Hence, for

$$\frac{2k_i}{2\pi/d_p} = \frac{2d_p}{\lambda_i} = n, \qquad \text{i.e. for} \qquad \frac{d_p}{\lambda_i} = \frac{n}{2}, \tag{17}$$

the integration is performed over an integer number of periods of a simple periodic function, the result being zero.

If we now consider the distribution $P_{MW}(\Delta k_x)$, described by (8), we obtain the transmission function [26]

$$T(y'_{12}, \Delta x_3) = a + bV \cos(2\pi \Delta x_3/d + d_p k_i), \tag{18}$$

where the visibility reads as

$$V = \frac{3}{4\pi} \frac{\lambda_i}{d_p} \left[\left(1 - \frac{1}{(2\pi)^2} \frac{\lambda_i^2}{d_p^2} \right) \sin\left(\frac{2\pi d_p}{\lambda_i} \right) + \frac{1}{2\pi} \frac{\lambda_i}{d_p} \cos\left(\frac{2\pi d_p}{\lambda_i} \right) \right]. \tag{19}$$

The zeros of (19) are very close to those of the visibility obtained from the constant distribution above (see figure 2 in reference [26]).

Finally, if we consider the distribution

$$P_I(\Delta k_x) = \gamma e^{-(\Delta k_x/N k_i)^2}, \tag{20}$$

where $\gamma = 2/N k_i \sqrt{\pi}$, we obtain

$$V = \frac{|\mathrm{erf}(2/N - i\alpha) + \mathrm{erf}(i\alpha)|}{\mathrm{erf}(2/N)}\; e^{-\alpha^2/4}, \tag{21}$$

$$\varphi = \frac{1}{2i}\ \ln\left[\frac{\mathrm{erf}(2/N - i\alpha) + \mathrm{erf}(i\alpha)}{\mathrm{erf}(2/N + i\alpha) + \mathrm{erf}(-i\alpha)}\right], \tag{22}$$

where $\alpha = Nk_i d_p$ and erf is the error function, $\mathrm{erf}(\alpha) \equiv \frac{2}{\sqrt{\pi}}\ \int_0^{\alpha} e^{-t^2} dt$. The visibility given by (21) does not have zeros; for any d_p/λ_i, this function is always above the values of the visibility arising from the previous two distributions. This is in agreement with the experimental data displayed in figure 2 of reference [16]. From this fact, Chapmann *et al* concluded [16] that *coherence was not really destroyed, but only entangled with the final state of the reservoir* (photons).

As can be seen, our results agree with the first part of the conclusions of Chapmann *at al* [16]. In evaluating the visibility, we have assumed that the scattering process does not destroy the atomic wave function (i.e. coherence), which describes the state of the atom before the scattering process, but only induces a certain shift and change of its phase. The visibility dependence on d_p/λ_i is thus a consequence of the statistical distribution of transferred momenta to the atom during photon-atom scattering process. Through the atom selection, which is equivalent to varying the distribution of transferred momenta, the visibility can change substantially. Within our approach, we have not invoked entanglement with the reservoir states; rather, we have determined and used the evolution of an atomic wave function before and after photon-atom scattering events.

Finally, we would like to stress that experimentally photon-atom scattering events take place at distances from a grating within the Fresnel region. This can be easily seen by means of the relation

$$y'_{12} = \frac{kd}{2\pi}\, d_p = \frac{d_p}{\lambda_i}\frac{kd}{k_i} = \frac{d_p}{\lambda_i}\frac{L_T}{2}\frac{\lambda_i}{d}. \tag{23}$$

In the experiment, the ratio d_p/λ_i goes from 0 to 2. By the values of the other parameters given in the caption of figure 1, it follows that $y'_{12} \in [0, 19.09]$ mm, with $L_T = 6.48$ mm.

4. Conclusions

To explain atom interference experiments with presence of photon-atom scattering processes, in our opinion it is necessary to use the atom wave function as well as to take into account its particle properties (i.e. the change of momentum during photon-atom scattering events). The experimentally established visibility dependence on d_p/λ_i was previously explained [26] by considering a random change of the atomic transverse momentum induced by the scattering with photons.

The experimental regain of visibility induced by selecting a subset of atoms from the set of all those transmitted through the third grating is explained by studying the visibility dependence on the probability distribution of transferred momenta. This atom selection does not provide any information about the place along the x-axis where the photon scatters from the atom. Consequently, it is not necessary to attribute the decrease and disappearance of visibility as d_p/λ_i increases to an increase of an observer's (potential) knowledge about the atomic path behind the first grating.

From our description, we also find that photon-atom scattering processes happen within the Fresnel region, where the atomic wave function has a very complex form (see figure 1). But, as has been shown within the context of the Talbot effect [14], the topology of the trajectories also becomes very complex within this region. Therefore,

not two but many atomic paths exist near the grating, where the photon hits the atom. As one moves further away from the grating (towards the Fraunhofer region), those numerous trajectories basically group along three main paths. Here, we have considered two of them for a chosen value of the transferred momentum to explain the experiment.

The agreement between our theoretical expressions for the visibility and the experimental curves thus supports, in our opinion, the views of Einstein, de Broglie, Bohm and others, i.e. that individual micro-objects can be simultaneously described by a wave and a particle. In particular, here we have stressed that such an agreement has been obtained by using both the space and momentum atom distributions as quantum objects. In this way, our results also agree with and support the more general conclusions due to Ballentine [30] and Khrennikov [31, 32]. According to these authors, the question of complementarity versus compatibility of wave and particle properties is tightly connected to the problem of the existence of a joint distribution of two dynamical variables associated with two non-commutative operators, as well as to the question of whether simultaneous measurements of two non-commutative operators is possible. In our theoretical approach, we have not considered a joint distribution of coordinates and momenta, but we used both distributions to explain the experimental results.

To recapitulate, our results show that different outcomes of the experiment should not be associated with the presence of an external observer, but with the sensitivity of the experiment which is being carried out or, in other words, the particular measurement performed. More specifically, with our analysis we have shown here that the visibility depends on the experiment itself and can be nicely explained taking into account all the elements present in such an experiment.

The importance of context, i.e. the concrete specification of the experimental setup in the analysis of interference phenomena is reconsidered by Ballentine [30] and Khrennikov [31, 32] from a general point of view. According to the latter author, the main structures of the Quantum Theory are already present in a latent form within the classical Kolmogorov probability model. In this regard, a very interesting question arises, namely to compare our findings and method with such an approach. At present, this idea is being considered as a direction for future research.

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